LATTICE FIELD THEORY 5:

Costs of Fermion Algorithms

Chiral Extrapolations

Staggered Fermions

Outlook

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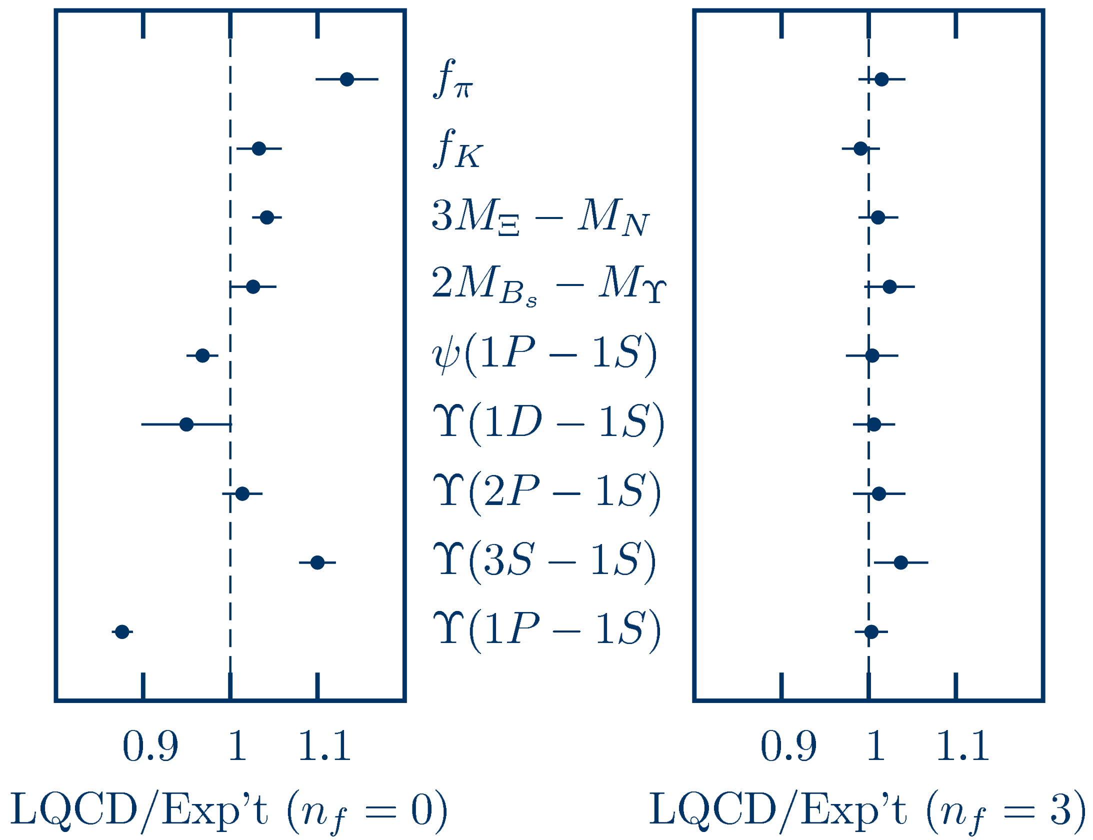
Lattice QCD with Improved Staggered Quarks

In the first lecture, we saw this plot $\Longrightarrow$

Set $1+4$ free parameters with $1+4$ meson masses.

Quenched (on left) shows discrepancies as much as 10–15%.

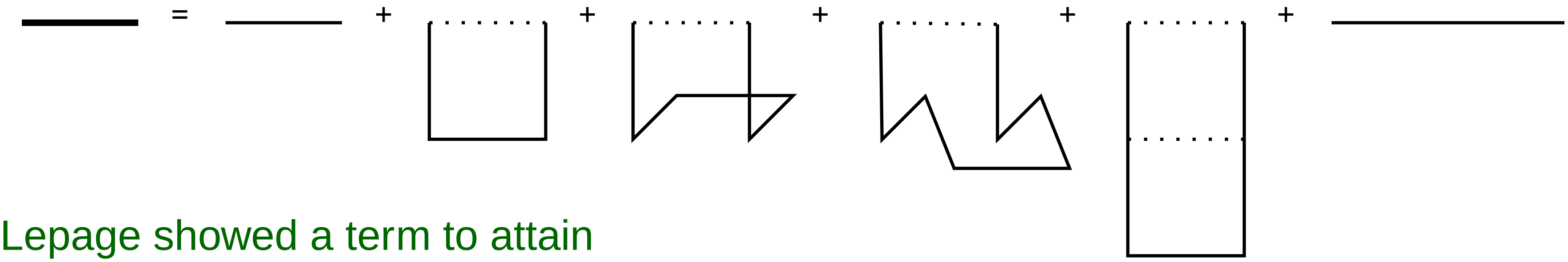
Unquenched QCD (on right) shows discrepancies of a few %. Within the error bars.



Davies *et al.*, hep-lat/0304004

Several developments led up to these results.

Toussaint and Orginos found an empirical way—Fat7—to remove the largest discretization effects (of order a^2) of staggered fermions—taste-changing interactions.



Lepage showed a term to attain Symanzik improvement $\rightarrow O(\alpha_s a^2, a^4)$—Asqtad.

The MILC Collaboration started a long project with $n_f = 2+1$ full QCD; they found they could go to quark masses as low as $0.15 m_s$.

MILC made the ensembles of gauge fields freely available. Physicists at Cornell, Fermilab, Simon Fraser, Glasgow, Illinois, & Ohio State starting working on them.

Aubin and Bernard's chiral perturbation theory with taste symmetry breaking.

Costs of Dynamical Fermions

Studies of algorithms for (improved) Wilson fermions suggest

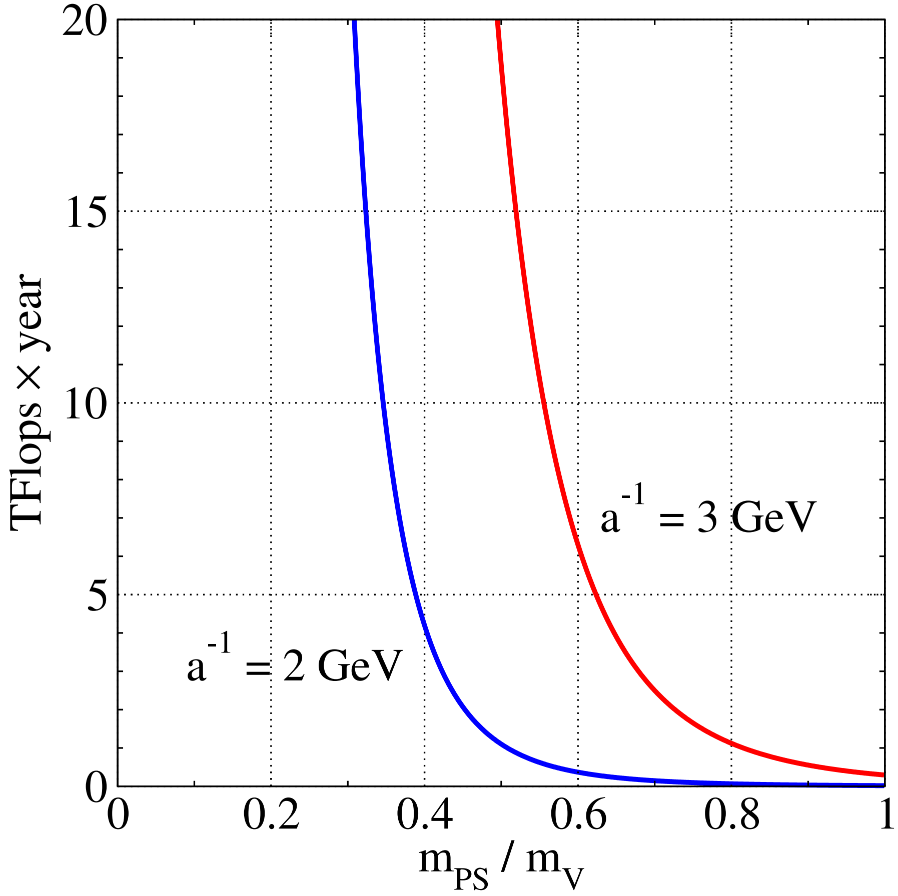
$$\mathtt{cost} \propto \left(\frac{m_{\mathrm{PS}}^2}{m_{\mathrm{V}}^2}\right)^3 L^5\, a^{-7}$$

The Berlin Wall.

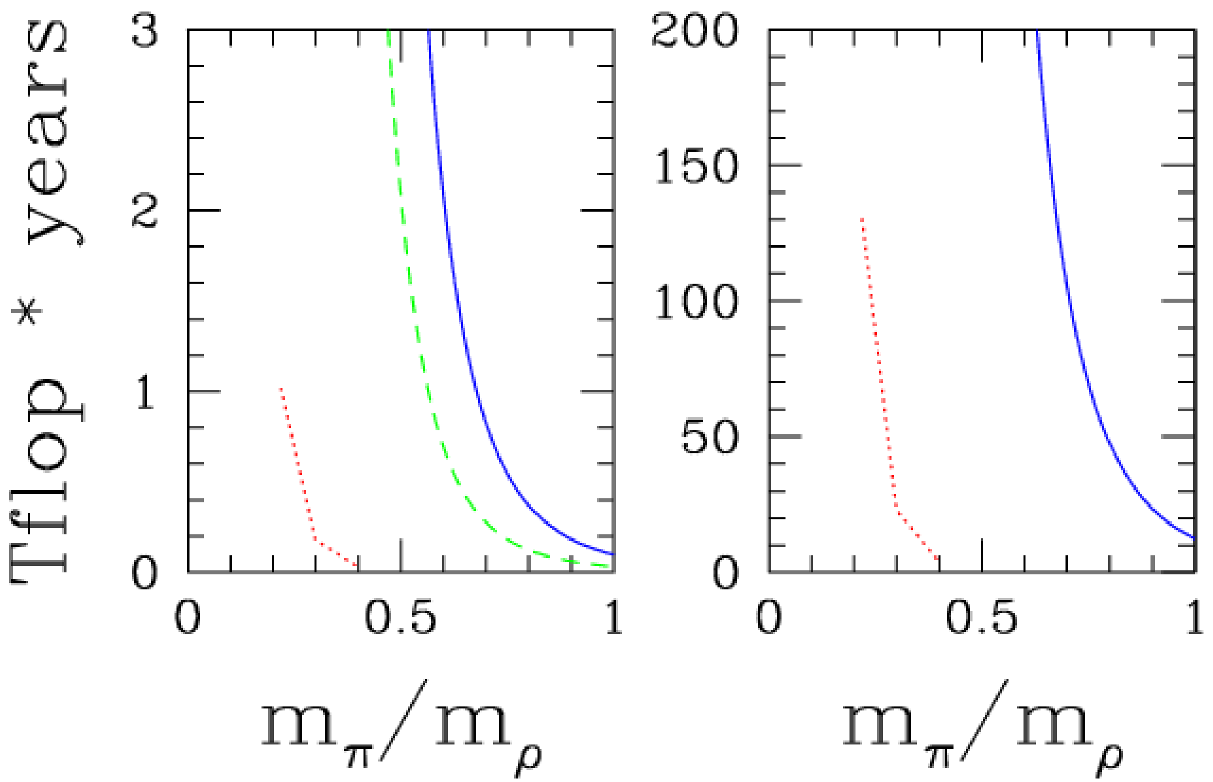
Phenomenology, not data.

1000 configurations with L=2fm

[Ukawa (2001)]



Staggered Quarks are Faster!



Ukawa | $a = 1/11$ fm | $a = 1/22$ fm
Ukawa/3 | measured | extrapolated

The Problem of Light Quarks

The algorithms literally hit a wall when

$$
\begin{array}{ll}
m_{\mathrm{PS}} < 0.7 m_{\mathrm{V}} & \text{Wilson (and SW)} \\
m_{\mathrm{PS}} < 0.3 m_{\mathrm{V}} & \text{staggered}
\end{array}
$$

or

$$
\begin{array}{ll}
m_q < 0.6 m_s & \text{Wilson (and SW)} \\
m_q < 0.1 m_s & \text{staggered}
\end{array}
$$

In Nature $m_d = 0.04 m_s$ and m_u is about 3 times smaller still.

Extrapolate $m_q \to 0$ (nearly), called the chiral extrapolation.

Often the largest source of systematic uncertainty (and frequently underestimated in quenched calculations).

Chiral Perturbation Theory

Chiral perturbation theory (χPT) is a systematic method to compute the dependence of hadronic quantities on the masses of the light pseudoscaler mesons:

$$A = A_0 + A_1(\mu)\frac{m_\pi^2}{(4\pi f_\pi)^2} + A_\chi \frac{m_\pi^2}{(4\pi f_\pi)^2} \ln(m_\pi^2/\mu^2)$$

The last term is called a "chiral log". $f_\pi = 132$ MeV

Really the limiting behavior of the function obtained from 1-loop integrals.

Something non-analytic in $m_\pi^2 \propto m_q$ always appears; not always a log

e.g., $m_\pi^3 = (m_\pi^2)^{3/2}$ in masses of heavy hadrons.

Replace m_π with m_{PS}, the mass as calculated in the simulation, and fit.

Chiral symmetry constrains A_χ to something known or "knowable." It is not a completely free parameter.

A Relevant Example

B-physics experiments measure the frequency Δm_d for neutral B_d mesons to oscillate between B^0 and $\overline{B}^0$. This is proportional to something called $f_{B_d}^2 B_{B_d}$.

The (much faster) oscillation frequency Δm_s for B_s mesons is expected to be measured "soon". $\hookrightarrow f_{B_s}^2 B_{B_s}$

Often assumed that "most of the theoretical uncertainty cancels" in

$$\xi^2 = \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}}$$

Except the uncertainty from chiral extrapolation.

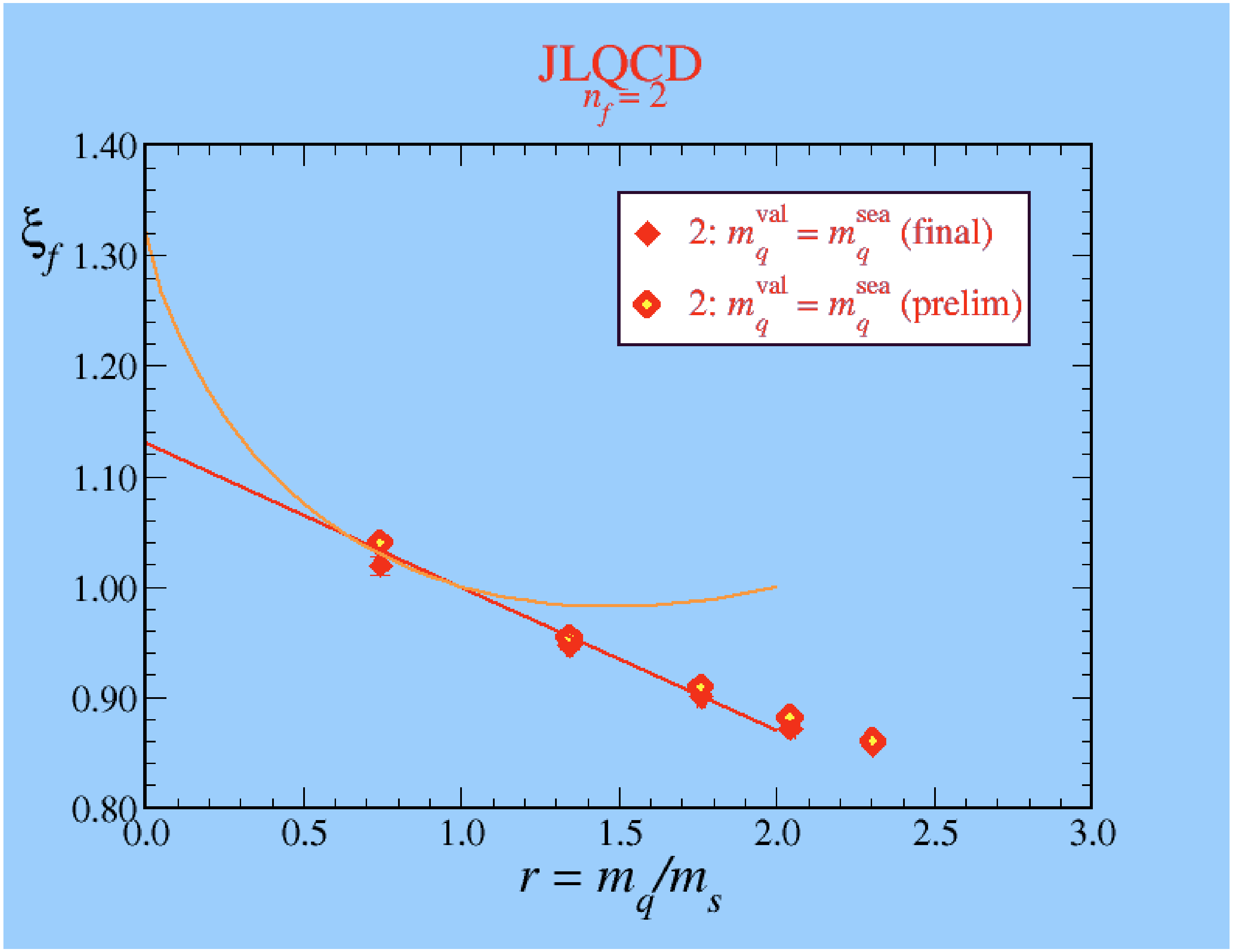
Even with $\Delta m_s < 15\ \mathrm{ps}^{-1}$, the most important constraint on CKM from lattice QCD.

$$\xi = \begin{cases} 1.30 \\ \qquad\qquad ??? \\ 1.13 \end{cases}$$

Best result

w/o staggered

light quarks



$$\xi = \begin{cases} 1.30 \\ \quad ??? \\ 1.13 \end{cases}$$

Result with

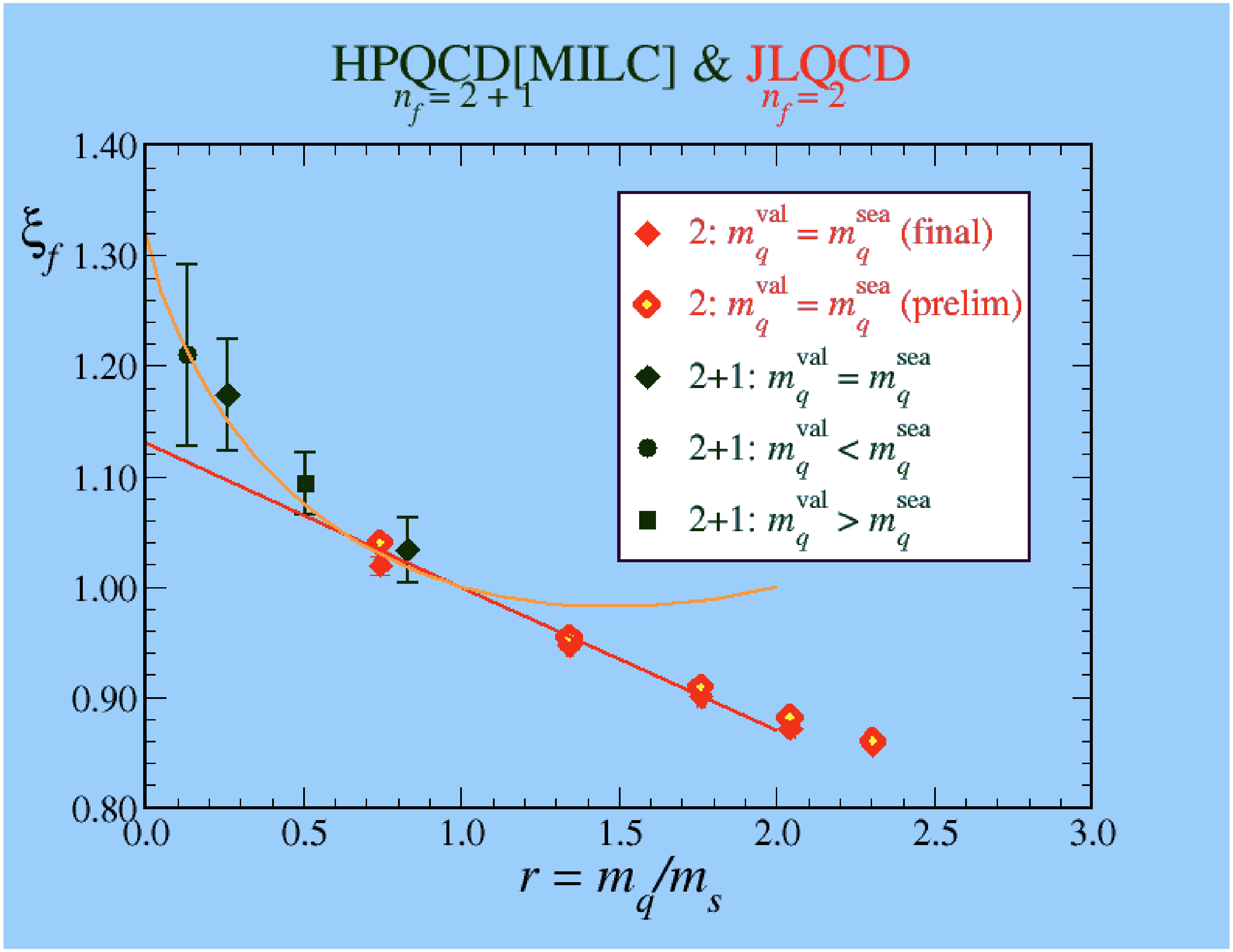
Asqtad $2+1$

[Wingate]

Best result

w/o staggered

light quarks



Problems with Staggered Fermions

Recall that staggered fermions come in 4 tastes.

How do we arrive at 2 flavors with one mass, and 1 flavor with a smaller mass?

$$\det M = e^{\operatorname{tr}\ln M} \quad \Rightarrow \quad \frac{\partial \operatorname{tr}\ln M}{\partial U} = \operatorname{tr}\left(M^{-1}\frac{\partial M}{\partial U}\right) = \sum_c \xi_c^\dagger M^{-1}\frac{\partial M}{\partial U}\xi_c,$$

where ξ is a vector of random numbers.

Multiply last expression by $n_f/4$, $n_f = 2, 1$.

$$\tfrac{1}{4}n_f \sum_c \xi_c^\dagger M^{-1}\frac{\partial M}{\partial U}\xi_c \qquad \Rightarrow \qquad \exp\left(\tfrac{1}{4}n_f \operatorname{tr}\ln M\right) = (\det M)^{n_f/4}$$

M is a discretization of $\not{D} + m$, for 4 tastes. Certainly $(\not{D} + m)^{n_f/4}$ is non-local and would have to be rejected.

2+1 Quarks for Muster Mark?

Staggered fermions are supposed to yield 4 Dirac fermions in the continuum limit.

There should be a basis

$$M = \begin{pmatrix} \tilde{M} & & & \\ & \tilde{M} & & \\ & & \tilde{M} & \\ & & & \tilde{M} \end{pmatrix} + a\mathcal{N} = \mathcal{M} + a\mathcal{N}, \qquad \mathcal{N} \text{ not block diagonal}$$

If the $O(a)$ is indeed small, then

$$\det M = (\det \tilde{M})^4 \left[1 + a\,\mathrm{tr}(\mathcal{N}\mathcal{M}^{-1})\right],$$
$$(\det M)^{n_f/4} = \det \tilde{M} \left[1 + \tfrac{1}{4} n_f a\,\mathrm{tr}(\mathcal{N}\mathcal{M}^{-1})\right],$$

which at least is not as non-local as $(\not{D} + m)^{n_f/4}$.

Further analysis is not so straightforward.

Explicit constructions of the 4 tastes are notationally voluminous.

The separation is clean (and local) for free fields, but muddied when gauge fields are introduced.

The Asqtad action was specifically designed to reduce taste-changing interaction, denoted here as $\mathcal{N}$.

At present there is circumstantial evidence. For me it is compelling enough to believe that this method should be pursued.

Chiral Extrapolations of Decay Constants

Dots at 0.04 are experiment values.

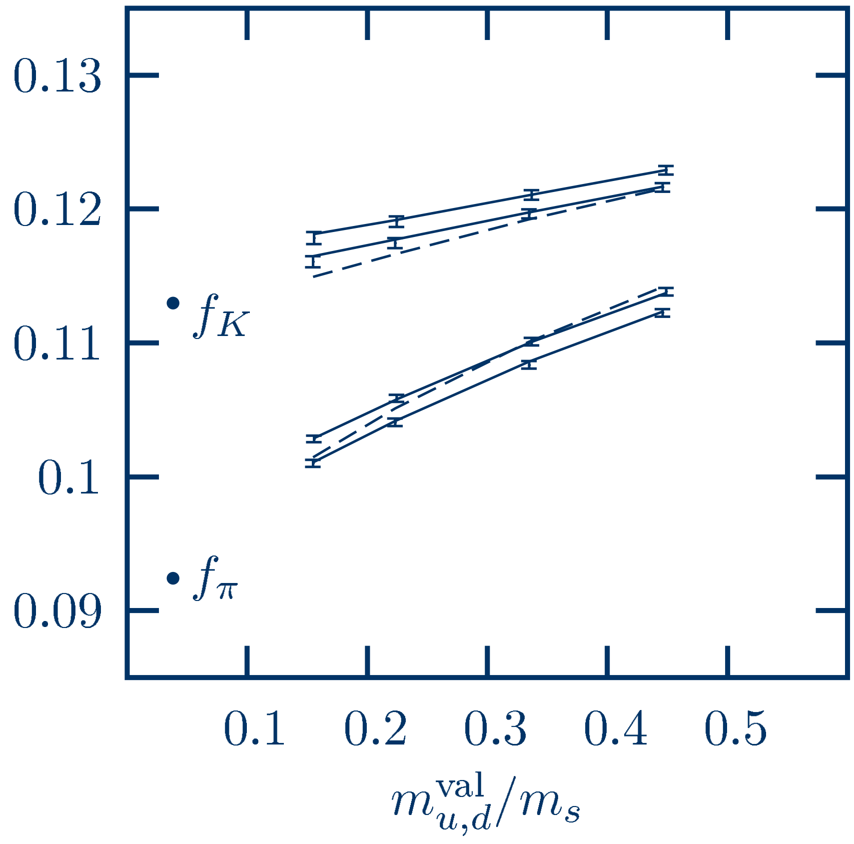
Error bars are lattice QCD.

Linear extrapolation (by eye) gets close.

A chiral log fit gets closer

Correcting for $O(a^2)$ gets even closer. (On the ratio plot.)

An even more remarkable analysis [Aubin & Bernard] follows.



Davies *et al.*, hep-lat/0304004

χPT for Taste-Symmetry Violation

WARNING: this gets complicated!

For 4 species the taste symmetry group should be SU(4)$\times$SU(4).

Discretization break it to $\Gamma_4 \times$U(1), leading to more non-analytic contributions in χPT.

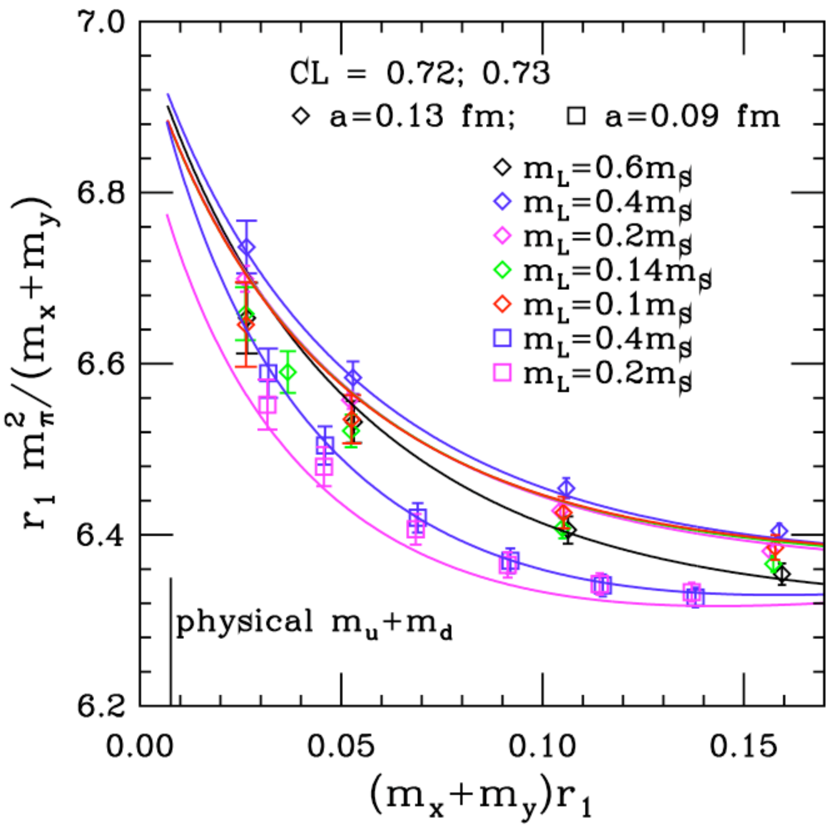
Also possible to account for $(\det M)^{n_f/M}$ in χPT: SU$(4|4-n_f)\times$SU$(4|4-n_f)$.

Ane possible to account for $m_q^{\text{valence}} \neq m_q^{\text{sea}}$.

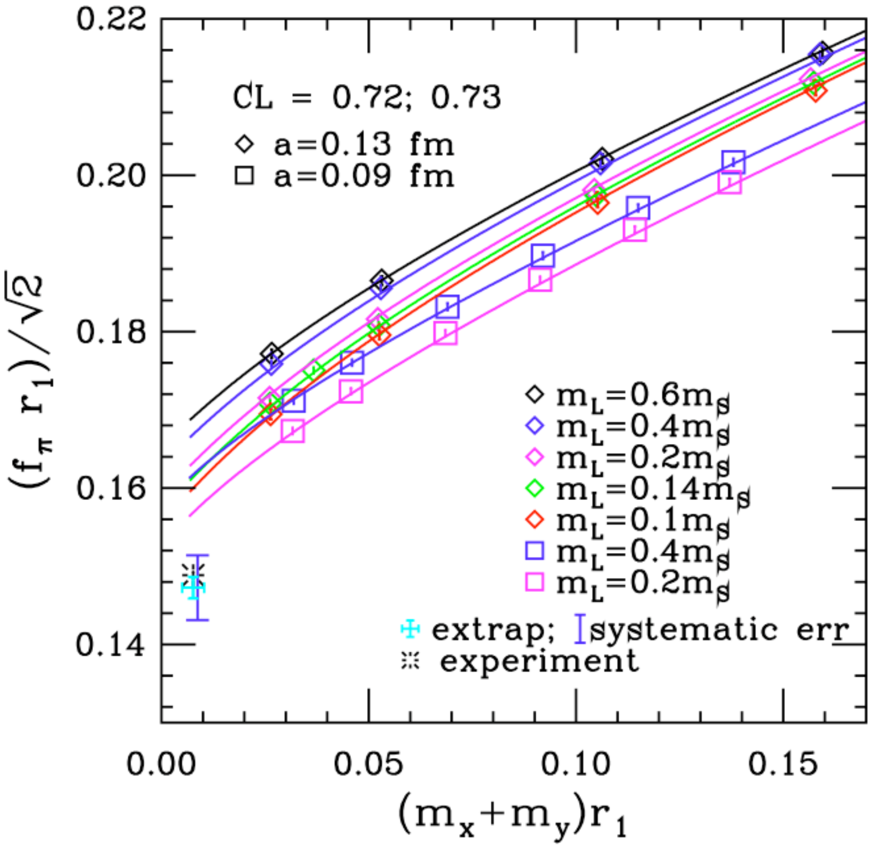
Aubin and Bernard put this all together to obtain unpresentable formulas.

Statistical precision of MILC is good enough to fit them.

χPT with Violations of Taste Symmetry



one fit



one fit

Outlook

The recent results with improved staggered quarks are very promising.

The goals are, however, extremely ambitious: uncertainties not merely small but robust enough to support a claim of new phenomena in B physics (if indeed it's there).

Any numerical simulation is, in the end, fairly inscrutable to outsiders. Are there any predictions? Any tests?

There are several things that should be easy for us (and are in progress), and, though unmeasured or poorly measured, will be measured well soon.

Decay constants and form factors of D and D_s mesons (coming from CLEO-c); the mass of the B_c (coming from CDF).

An especially intriguing test is as follows.

CLEO-c will measure $D \to l\nu$ and $D \to \pi l\nu$. The ratio

$$\frac{1}{\Gamma(D \to l\nu)} \frac{\Gamma(D \to \pi l\nu)}{dE_\pi} \propto \left[\frac{|f_+(E_\pi)|}{f_D}\right]^2$$

is a direct test of non-perturbative QCD. The missing factor is simply kinematics. Couplings such as CKM and even G_F drop out.

similarly $|f_+(E_K)|/f_{D_s}$

Next year I hope I can plot lattice QCD and (a few months later) overlay experiment.